The Energy Cost of Location Information in Wireless Sensor Networks

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Abstract—Based on a study of the optimal tracking error, we investigate the energy cost of obtaining location information in emerging wireless sensor networks. Specifically, we study the case of signal strength based positioning and tracking under the assumption of log normal shadowing. We find that for node densities in which signal strength positioning is viable, an a priori determination of the path loss exponent of the shadowing model is energy inefficient and unnecessary in terms of location accuracy. We also find that accuracy improvements in tracking obtained by increasing the radio transmission range of the sensor nodes is energy inefficient in relation to accuracy improvements. Based on our results we propose a simple location-update protocol that designers of location-based network functionalities should deploy.

Index Terms—Location, tracking, sensor networks.

I. INTRODUCTION

POSITION location and tracking is of growing interest for many applications in wireless networks. In many emerging wireless networks, such as wireless sensor networks, WiFi networks and wireless mesh networks, the Received Signal Strength (RSS) values, as measured by reference nodes of known position, can be used as a source of positioning information. In emerging wireless sensor networks [12], RSS-based positioning is likely to predominate due to the high-density of the nodes and the simplicity of obtaining RSS information. RSS-based positioning can be based on an a priori measured database of expected RSS values as a function of position [1], or on the use of propagation models to predict the RSS values as a function of distance e.g. [10] [4] [3]. The latter of these scenarios, and the one we study here, is required for situations where the sensor network has been rapidly deployed, the radio environment is unstable, or (of course) where no a priori RSS database exists.

The particular focus of this work is the energy cost of obtaining accurate location information. The energy budget of acquiring location information in the context of location-based network functionality, such a geographical routing, has been investigated before e.g. [8] [9]. What has been missing from the literature is an assessment of the energy costs required to meet a specific position accuracy. It is this gap in knowledge that we address here. Our specific contributions are (i) we use optimal tracking performance arguments to determine the minimum energy requirements required by a wireless sensor network to meet specific accuracy requirements; (ii) we probe the importance of ignorance on the propagation model parameters on these energy requirements; and (iii) based on our results we propose a simple “update-location” protocol that is energy efficient.

II. ANALYSIS OF THE ENERGY COST OF LOCATION

A. Optimal Tracking in Wireless Networks

The fundamental error bounds for tracking in a wireless network are best described in terms of the Cramer-Rao Lower Bound (CRLB) on the error variance of a mobile node position as it traverses the network area. Determination of the CRLB which takes into account tracking information is substantially more complex than CRLB determination for the static case. In previous works, tracking (or dynamic) CRLB bounds have been derived for the generic case e.g. [2] [14] [6], as well as for specific cases where time difference of arrival is the specific information metric utilized [15]. A recent review of this area of research in the context of FCC (Federal Communications Commission) regulations can be found in [4]. Here we expand these ideas to the case of RSS under the assumption of a log normal shadowing model appropriate to wireless sensor networks. We will also include in our analysis, the situation where the key parameter of the propagation model - the path loss exponent - is also an unknown. This represents the realistic situation where the propagation model of the environment, in which the sensor network is deployed, has not been measured or studied in any detail.

In the discrete-time tracking problem with additive Gaussian noise the evolution of the state vector $p$ as a function of the time-step $k$ can be written

$$p_{k+1} = f_k(p_k) + q_k,$$  \hspace{1cm} (1)

where $q_k$ represents zero-mean Gaussian process noise with covariance matrix $Q$ (which we assume to be non-singular), and where the evolution function $f_k$ can in general be non-linear. Most tracking filters operate by adjusting the predicted state vector given by Eq. (1) with an update determined from incoming measurements $z_k$. Such measurements are related to the state vector through

$$z_k = h_k(p_k) + r_k,$$  \hspace{1cm} (2)

where $r_k$ represents zero-mean Gaussian measurement noise with covariance matrix $R$ (again assumed non-singular), and where $h_k$ is the function relating the observation to the state vector. Again, in general, this latter function will be non-linear.
The work of [14] (see also [2] [13]) outline how the Fisher information matrices, $F_k$ $(k > 0)$ of the tracking problem can be written as a Riccati-like recursion:

$$F_{k+1} = D_k^T - (D_k^T) [F_k + D_k^T]^{-1} D_k^T + F_k (k+1) \quad (3)$$

where under the assumption of additive process noise, $D_k^T = E \{ J_k^T Q_k^{-1} J_k \}$, $D_k^T = -E \{ J_k^T \} Q_k^{-1}$, $D_k^T = Q_k^{-1}$, where $J_k$ is the Jacobian of the state evolution function, and $E \{ \cdot \}$ denotes expectation. The last term of Eq. (3) provides the dependency of the CRLB on the measurement model and is given explicitly in terms of probability densities $g$ by

$$F_k (k+1) = E \left\{ \left[ \nabla p_{k+1} \ln g(z_{k+1}|x_{k+1}) \right] \times \left[ \nabla p_{k+1} \ln g(z_{k+1}|x_{k+1}) \right]^T \right\}. \quad (4)$$

In the tracking problem we will consider here - the position determination of a wireless sensor node - the state evolution function will be taken as the linear form $f_k(p_k) = p_k + T_s v_k$, where $v_k$ is a potential drift velocity, and $T_s$ is the sampling time. In all our analysis and relations we will adopt $T_s = 1$ (and therefore drop its explicit appearance). The linear evolution model can be sub-classed into the random motion ($v_k = 0$) or the linear velocity sensor model ($v_k \neq 0$). In most tracking situations in wireless sensor networks such linear evolution models will suffice.

Under the assumption of a random walk motion model, or a velocity sensor model, determination of the $D_k$ terms of Eq. (3) is trivial, and one can show from Eq. (3) and Eq. (4) that the one step-ahead prediction for the covariance matrix $P$ (of the estimation error on the state vector) is

$$P_{k+1} = \left( (p_k + Q_k)^{-1} + F (p_k^0) \right)^{-1}. \quad (5)$$

The term $F (p^0)$ represents the static Fisher information matrix (the details of which we describe below) at the current true state $p^0$. Under some mild assumptions on the nature of the measurement function, one can show that the stationary solution of this equation is given by [4]

$$\hat{P} = -\frac{1}{2} Q + F^{-1/2} (p^0) \times \left\{ F^{1/2} (p^0) [Q + QFQ] F^{1/2} (p^0) \right\}^{1/2} F^{-1/2} (p^0). \quad (6)$$

These relations determine the fundamental performance bound on the position of a wireless sensor node. In the simulations to be discussed later, we find that the optimal tracking error predicted by Eq. (6) is to all intents and purposes indistinguishable from that predicted by Eq. (5).

To proceed we must now determine the matrix $F (p^0)$ under the assumption of log-normal shadowing. In the log-normal shadowing model the Path Loss (PL) at a distance $d$ is written

$$PL(d)[dB] = PL(d_0) + 10 n log_{10} \left( \frac{d}{d_0} \right) + X_{\\sigma}, \quad (7)$$

where $X_{\\sigma}$ is a zero-mean Gaussian distributed random variable (in dB) with standard deviation $\sigma_d$, $n$ is the path-loss exponent, $d_0$ is a receiver reference distance, and the bar indicates an ensemble average. The path loss term $PL(d)$ is related to the RSS measured at a distance $d$ through

$$P_r(d)[dBm] = P_t(d)[dBm] - PL(d)[dB], \quad (8)$$

where $P_r(d)$ and $P_t(d)$ are the received and the transmitted power, respectively (in dBm). Writing the signal strength (in dB) as $S = 10 log_{10}(P_r(d)/P_t(d_0))$, the above relations lead to a probability density

$$g(S) = \frac{1}{\sigma_d \sqrt{2\pi}} \exp \left\{ - \left[ S + n ln \left( \frac{d_0}{d} \right) \frac{10}{2 \sigma_d^2} \right]^2 \right\}. \quad (9)$$

These relations describe our measurement model to be used in the tracking CRLB determination (and also in the Extended Kalman Filter (EKF) algorithm that we will use). It is evident from the above relations that the noise covariance matrix $R$, at a given position, associated with this observation model can be described in terms of a zero-mean normal probability distribution with variance $\sigma_d^2$.

From Eq. (9) we can calculate the terms of the Fisher matrix through

$$F_{ij} = -E \left\{ \partial^2 \ln g(S) \right\}, \quad (10)$$

where the indexed $i$'s represents all the unknown elements of the state vector, some of which we may not be interested in (such as $n$). In practice the path loss exponent $n$ will be undetermined unless there has been an a priori study of the sensor nodes environment, and that the propagation model is constant in that environment. In general neither of these circumstances are normally true. To account for this we will treat $n$ as a nuisance parameter. Although we do not need to determine the value of such nuisance parameters, they will influence the accuracy of our position estimation - which is what we are ultimately interested in. We discuss below the cost of obtaining the value of $n$ in an independent measurement campaign, and compare that cost with the loss in position accuracy suffered by treating $n$ as a nuisance parameter whose value we do not know a priori. A discussion of the less important parameter $\sigma_d$ in relation to CRLB’s is given in [7]. Although $\sigma_d$ has an influence on accuracy, it is not a nuisance parameter (it would not enter into any positioning algorithm explicitly).

Assuming planar motion and $N$ reference nodes, the Fisher matrix can be written

$$F = \left[ \begin{array}{ccc} \alpha & \gamma & \gamma \\ \gamma & \beta & \gamma \\ \gamma & \gamma & \beta \end{array} \right], \quad (11)$$

where the elements of this matrix are given by [7]

$$\alpha = b \sum_{i=1}^{N} \left\{ \frac{\cos^2 \phi_i}{\sin^2 \phi_i} \left[ 1 - \frac{1}{\sin^2 \phi_i} \ln \left( \frac{d_{i0}}{d} \right) \right] - \frac{1}{\sin^2 \phi_i} \ln \left( \frac{d_{i0}}{d} \right) \sum_{j=1, j\neq i}^{N} \cos \phi_i \cos \phi_j \ln \left( \frac{d_{ij}}{d_{ij}} \right) \right\}, \quad (12)$$

$$\beta = b \sum_{i=1}^{N} \left\{ \frac{\sin^2 \phi_i}{\sin^2 \phi_i} \left[ 1 - \frac{1}{\sin^2 \phi_i} \ln \left( \frac{d_{i0}}{d} \right) \right] - \frac{1}{\sin^2 \phi_i} \ln \left( \frac{d_{i0}}{d} \right) \sum_{j=1, j\neq i}^{N} \sin \phi_i \sin \phi_j \ln \left( \frac{d_{ij}}{d_{ij}} \right) \right\}, \quad (13)$$
and
\[ \gamma = \sum_{i=1}^{N} \left\{ \frac{\sin^2 \varphi_i}{2d_i^2} \left[ 1 - \frac{1}{S_i} \ln^2 \left( \frac{d_i}{d_0} \right) \right] - \frac{1}{S_i} \cos \varphi_i \ln \left( \frac{d_i}{d_0} \right) - \sum_{j=1 \land (j \neq i)}^{N} \frac{\sin \varphi_i \ln \left( \frac{d_i}{d_j} \right)}{S_i} \right\}. \]

Here
\[ b = \left[ \frac{10n}{\sigma_{db} \ln 10} \right]^2, \]
\[ S_n = \sum_{i=1}^{N} \ln^2 \left( \frac{d_i}{d_0} \right), \]
\[ \sin \varphi_i = \frac{x_i - x_0}{d_i}, \]
\[ \cos \varphi_i = \frac{y_i - y_0}{d_i}, \]
and \( d_i \) refers to the distance from the device of unknown position \((x_0, y_0)\) to the node of known position \((x_i, y_i)\).

The trace of the inverse of the matrix given in Eq. (11), then provides the static CRLB bound on the variance of the position error of the node in the 2 dimensional plane. In the case where the path-loss exponent \( n \) is known \textit{a priori} the algebra simplifies and one can show, e.g. [10] [7], that the static CRLB is
\[ \nu_{(n \; \text{known})} = \frac{\sum_{i=1}^{N} \frac{1}{d_i^2}}{(\frac{10n}{\sigma_{db} \ln 10})^2} \left( \frac{1}{N} \sum_{i=1}^{N} \sum_{j=i+1}^{N} \frac{\sin^2 \varphi_i \ln \left( \frac{d_i}{d_j} \right)}{S_i} \right). \]

The above relations taken together detail how to determine the dynamic CRLB. They do not provide for an algorithm to actually track the mobile node. Determination of tracking algorithms which have a performance approaching the dynamical CRLB is not the thrust of this specific work. However, it will be perhaps useful in our simulations to track the mobile node using a standard EKF algorithm (in order to determine EKF performance relative to the CRLB).

To construct the EKF for log-normal fading we must first construct various Jacobian matrices related to the motion and observation models described above. Defining \( H \) as the Jacobian matrix of partial derivatives of the observation model with respect to \( p \), \( V \) as the Jacobian matrix of partial derivatives of the observation model with respect to \( X \), \( A \) as the Jacobian matrix of partial derivatives of the motion model with respect to \( p \), and \( W \) as the Jacobian matrix of partial derivatives of the motion model with respect to \( q \); the EKF is then implemented as (in these steps the minus superscript indicates a priori state estimates): (i) Project state ahead \( p_k = f_k(p_{k-1}, v_k) \); (ii) Project error covariance ahead \( P_k = A_f P_{k-1} \cdot A_f^T + W_k P_{k-1} W_k^T \); (iii) Compute Kalman gain \( K_k = P_k H_k^T (H_k P_k H_k^T + V_k)^{-1} \); (iv) Update estimate with RSS measurement from \( i \)th reference node \( p_k = p_{k-1} + K_k (RSS_i - RSS_{i,\text{pred}}(P_k)) \); and (v) Update the error covariance \( P_k = (I - K_k H_k) P_{k-1} \).

B. Energy Cost Function

The mechanisms and protocols for providing location updates in mobile wireless networks are many and varied e.g. [8] [9]. However, the general principles remain much the same in almost all implementations. In general, mobile nodes transmit signals to some nearby reference nodes of known position. The reference nodes, upon receipt of the signal from the mobile node, will transmit their location information back to the mobile node. The mobile node will use this information in conjunction with its RSS measurements to determine its location.

The problem is to ascertain how accurately we can track a mobile node as it traverses a region of reference nodes, and what is the energy cost of attaining the mobile node’s position as a function of position accuracy.

To provide focus we will select a specific location update scheme that has recently been presented in the literature. We will analyze the Neighborhood Discovery Protocol (NDP) discussed in [8]. This location information protocol operates very much in the way we have just described above. The mobile node emits Neighborhood Discovery packets which are received by all reference nodes within radio range. The receiving reference nodes then transmit Location Update packets (containing the reference node position) back to the mobile node. A cost analyses of this scheme can be described as [8]
\[ C = \left[ L_D \beta \lambda^n + (N_r \lambda + 1) L_D E_{\text{elec}} + \sum_{m \in \xi(k)} L_U \beta \lambda^m + 2 N_r \lambda L_U E_{\text{elec}} \right] \frac{1}{T_m} \]

with \( n \) again being the path loss exponent; \( \beta \) a constant in (Joule/(bits \( d_u \))) \( (d_u \) is unit length); \( L_D \) the length of the Neighborhood Discovery packet (in bits); \( L_U \) the length of the Location Update packet (in bits); \( E_{\text{elec}} \) the energy (in Joules) needed by the transceiver circuitry to send or receive one bit; \( N_r \lambda \) the number of reference nodes within range \( \lambda \) of the mobile node; \( \xi(k) \) the set containing the reference node indices within range \( \lambda \); \( \lambda_m \) the distance between a reference node \( m \) and the mobile node; and \( T_m \) the period between two consecutive Neighborhood Discovery packets (in secs).

In effect Eq. (16) details the energy cost to send out the Neighborhood Discovery packets over a range \( \lambda \) plus the receipt cost of that packet by \( N_r \) receivers. The term under the summation sign represents the cost of all reference nodes within range sending back a Location Update packet over the individual ranges \( \lambda_m \) (in practical implementation this could be taken as \( \lambda \) for all nodes). A fuller description and derivation of Eq. (16) can be found in [8]. Although other schemes in the literature differ in some details, they will largely result in an energy cost analyses very similar to that described by Eq. (16). More specifically, the major points we draw later

1These reference nodes are assumed to have already attained their own accurate position information via embedded GPS or some other a priori external location information source. It is assumed that the reference nodes are also energy constrained and will not continually broadcast location update information. Note also, we will not discuss here the energy cost of CPU processing of the information metrics required to deliver a position. This processing may be done by the sensor nodes or handed off to a server unconstrained by energy requirements.
from our location-cost analysis will be similar regardless of the detailed protocol implementation.

The relation of Eq. (16) describes the cost of obtaining a single location update in units of Watts. We wish to study the ongoing cost of obtaining the location information as the node traverses the physical space of the sensor network. More specifically we wish to probe the trade-off between location accuracy and energy usage. Clearly increasing the radio range of the node will increase accuracy at the cost of additional energy. Quantifying this trade-off is what we determine next.

III. PERFORMANCE EVALUATION

We now have the analysis in place which allows us to simulate the energy cost of obtaining location information in a sensor network under the assumption of log-normal shadowing.

We will largely probe log-normal shadowing parameters in the range of 2 – 4 for \( n \), and 6 – 12 for \( \sigma_{db} \). Such values are anticipated in many environments, both indoor and outdoor [11]. The network density of nodes will be explored in the range of 0.5-3 per 10\( m^2 \). Such densities are typical of those anticipated for emerging wireless sensor networks [12]. In our simulations, the nodes all report a signal strength of \(-52\,\text{dBm} \) at 2\( m \). We adopt a sampling period of 1/s. Some of our motion models possess a drift velocity of 2\( m/s \). To this drift velocity we add random perturbations with the terms of \( Q \) bounded by \((3m/s)^2\). Given the typical size of wireless sensor networks this variance bound represents a reasonable upper limit on the mobility uncertainty of an out-of-vehicle mobile sensor node. Unless otherwise stated, the diagonal values of the variance matrices this variance bound represents a reasonable upper limit on the mobility uncertainty of an out-of-vehicle mobile sensor node. Unless otherwise stated, the diagonal values of the variance matrices of the radio transmitters.

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Our first simulation probes the influence of the nuisance parameter \( n \) in the energy cost vs. accuracy space. The top diagram in Fig. (1), shows a typical distribution of reference nodes, in which the true path (solid line) of a mobile sensor node is given. This path possessed a drift velocity of 2\( m/s \). The other path (dashed line) indicates the track predicted by our EKF algorithm. At a value of 20\( m \) for the radio transmitter range the EKF gave an average position error of 4\( m \). For small variances in \( Q \), improvement in tracking error was found to be consistent with that predicted by our earlier analysis. In Fig. (1) the path is tracked for a total time of 15 seconds, and values of \( n = 3 \) and \( \sigma_{db} = 6 \) have been adopted.

The bottom diagram of Fig. (1) shows the optimal tracking error predicted for the true path as a function of the range of the radio transmitters. The tracking error is given as the average over the true path of the mobile node. Various curves are shown. The top curve represents the static optimal error in the case where \( n \) is a priori unknown. Here no tracking information or motion model has been used in order to ascertain a location. The second from top curve is the same except that it is assumed that \( n \) is known a priori. The first important point to note is that these curves are almost indistinguishable. The next two curves of this diagram represent the cases \( n \) known and \( n \) unknown, respectively, where the dynamic CRLB has been used to determine the optimal tracking error. Again we see that these curves are effectively indistinguishable. In Fig. (2) the same calculations are given except that we have adopted \( n = 2 \) and \( \sigma_{db} = 12 \) in order to show the effect of different propagation parameters. We can see that although the predicted position error as a function of range has increased, the indistinguishability of the \( n \) known and \( n \) unknown curves remains.
Our main conclusion from the results shown in Fig. (1) and Fig. (2) is that the additional energy cost of attaining a priori knowledge of \( n \) would not be worth the position accuracy gains. Determination of the value of \( n \) would require the movement of a reference node over a similar track to the mobile node, with multiple measurements \( i \) required in order to bring the variance of \( n \) down to acceptable levels. The energy cost would then be \( i \) times the cost of determining a position without knowledge of \( n \). Given the small gain in accuracy this would clearly be a poor trade-off. We have explored a wider range of the parameter space beyond the results shown explicitly here, as well as Monte Carlo simulations of the node positions and paths taken. We find that our conclusion above is robust over most expected conditions. Only in the case where the reference node density becomes substantially smaller (< 0.2 per 10m²) does the influence of a priori knowledge on \( n \) lead to position accuracy gains of greater than 1m. However, such low densities would not be typical of sensor networks in which RSS-based positioning would be deployed (at such low densities a large position uncertainty greater than 10m would be obtained even with \( n \) known).

In Fig. (3) we explore in more detail the energy requirements for a given tracking accuracy. Here again the top diagram represents a typical path and a typical representation of node positions. The middle diagram represents tracking accuracy for the true path as a function of the range of the radio transmitter. As we have now established that it is not necessary to distinguish between \( n \) known and \( n \) unknown cases, only the curves for the \( n \) known case are shown. The bottom diagram shows the power cost in Watts in order to achieve a specific tracking accuracy. Since this power curve is based on optimal tracking it provides for the minimum energy requirements for a given accuracy. Curves are shown for two different values of \( E_{\text{elec}} \) - the energy required by transceiver electronics to transmit or receive 1 bit. Below \( E_{\text{elec}} = 50 \text{pJ/bit} \) we find that the power curves are not influenced by the actual value of \( E_{\text{elec}} \). This means the power required to transmit over a given range dominates the energy cost equation. The simulation in Fig. (3) adopts \( n = 3 \) and \( \sigma_{db} = 6 \). For comparison we show a similar simulation for \( n = 2 \) and \( \sigma_{db} = 12 \) in Fig. (4). We can see that the trends of power consumption are very similar except, as expected, the tracking error is larger in the case of Fig. (4). In addition, in Fig. (5) we show the results for a random motion model (zero drift velocity), again with similar trends in the power vs. accuracy curves found. Many more simulations over the parameter space, node position representations, motion models, and velocity noise space have been carried out. The results shown explicitly in Figs. (3)-(5) are a good representation of those other simulations.

The main conclusion we wish to draw from our power cost simulations is that increases in the optimal tracking accuracy is extremely expensive in terms of additional energy requirements. A gain of an extra 0.5m in accuracy can require two orders of magnitude increase in energy. This fact can be traced back to the inefficiency of RSS-based positioning when reference nodes are beyond a range of 20m (e.g. [7]). A review of our early analysis shows how the contribution of a reference node to the static CRLB will be dependent on the distances to the fourth power. This means nearby reference nodes have a much greater impact on the position accuracy. This effect carries through through the dynamic CRLB analysis to give the trends shown – consistent with expectations from Eq. (5).

Any analysis on the energy requirements of a specific location based network function, such as location-based routing, should take into account the cost of acquiring a specific location accuracy. The results provided here will be useful in this regard. In addition, the energy cost analysis we have provided should be considered by designers of position-update protocols. Based on our results we propose that all network functionality and update protocols should be designed with a position accuracy of order one third the average inter-node separation distance \( d_{ns} \), provided that \( d_{ns} < 20 \text{m} \). A simple energy efficient (SEE) protocol can be readily achieved on that basis. It would possess the following steps. (i) When a location update is required the mobile node should transmit at a power level sufficient to cover a range of \( d_{ns} \). (ii) Provided at least three reference nodes respond with update packets - determine a position location. (iii) On failure to receive at least three update packets, increase the power of the radio transmitter to cover \( 2d_{ns} \) and repeat step (ii). (iv) If at least three update packet still not received increase range to \( 3d_{ns} \), and so forth.

A simple analysis of the SEE protocol, based on the binomial distribution and the assumption of a uniform random distribution of node positions, leads to the conclusion that the probability of having to take step (iii) is small (and the need to take step (iv) vanishingly small). This fact, and an analysis of our static CRLB formalism, shows that position errors expected from three or more reference nodes leads to (on average) values of position error of order 0.5\( d_{ns} \). Only in very rare occurrences will geometrical effects conspire to negate this fact. Use of tracking can readily reduce this error
normal shadowing. We have found that, in most cases, an a priori determination of the path loss exponent of the shadowing model is energy inefficient given the marginal accuracy gains in position achieved. We have also found that accuracy improvements in tracking, obtained by increasing the transmission range of the nodes, is very inefficient. Based on these conclusions we have proposed a simple position update protocol that designers of location-based network functionalities could deploy.

We have focussed on the optimal tracking capabilities (and therefore minimum energy requirements) of wireless sensor networks. Detailed algorithm development to achieve these optimal bounds has not been a focus of the work reported here. Our standard EKF tracking algorithm, deployed as an indicator of tracking capability, was in general found to be 50% efficient relative to the dynamical CRLB. Future work should be aimed at improving upon this. Use of particle filter algorithms at achieving tracking performance approaching the dynamic CRLB forms part of our own ongoing efforts in this area. Early indications are that tracking algorithms close to the dynamic CRLB can indeed be developed for wireless sensor networks, making the minimum energy requirements we have determined here even more relevant.

**REFERENCES**


